

DO NOW

Answers: pg 245

18. (a) 0 (b) -2/3 (c) $-\infty$

20. (a) 0 (b) 5/4 (c) ∞

22. 1/3

24. 4

26. $-\infty$

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4.5 Limits at Infinity - Day 2

Find the horizontal asymptotes of the following rational functions.

1. $f(x) = \frac{x^2}{x^2 - 9}$
 $\lim_{x \rightarrow \infty} \frac{x^2}{x^2 - 9}$
 $y = 1$

2. $g(x) = \frac{x + 4}{x^2 - x - 20}$
 $\lim_{x \rightarrow \infty} \frac{x + 4}{x^2 - x - 20}$
 $y = 0$

3. $q(x) = \frac{x^2 - 4}{x + 1}$
 $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x + 1}$
 ∞
 None

4. $r(x) = \frac{5 - 2x}{x}$
 $\lim_{x \rightarrow \infty} \frac{5 - 2x}{x}$
 $y = -2$

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Limits involving Trig Functions:

$\lim_{x \rightarrow \infty} \sin x$
D.N.E.

$\lim_{x \rightarrow \infty} \frac{\sin x}{x}$
 $\frac{\sin \infty}{\infty}$
 $-1 \leq \sin \infty \leq 1$
 ∞
 Squeeze Theorem
 0

Find each limit:

5. $\lim_{x \rightarrow \infty} x^3$
 ∞

6. $\lim_{x \rightarrow -\infty} x^3$
 $-\infty$

7. $\lim_{x \rightarrow \infty} \frac{3x^3 - 5x}{x^2 + 1}$
 $\lim_{x \rightarrow \infty} \frac{3x - \frac{5}{x}}{1 + \frac{1}{x^2}}$
 $\frac{3(\infty) - 0}{1 + 0}$
 ∞

8. $\lim_{x \rightarrow -\infty} \frac{3x^3 - 5x}{x^2 + 1}$
 $\lim_{x \rightarrow -\infty} \frac{3x - \frac{5}{x}}{1 + \frac{1}{x^2}}$
 $\frac{3(-\infty) - 0}{1 + 0}$
 $-\infty$

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9. Evaluate the limits of the function at ∞ and $-\infty$.

$f(x) = x^5 + 3x^2 - 8x + 5$

$f(x) = x^5 \left(1 + \frac{3}{x^3} - \frac{8}{x^4} + \frac{5}{x^5} \right)$
 $\lim_{x \rightarrow \infty} (\infty)^5 (1 + 0 - 0 + 0)$
 ∞

$\lim_{x \rightarrow -\infty} (-\infty)^5 (1 + 0 - 0 + 0)$
 $-\infty$

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If $x \geq 0$, then $x = \sqrt{x^2}$
If $x < 0$, then $x = -\sqrt{x^2}$

10. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$
 $\lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2 + 1}}{\sqrt{x^2}}}$
 $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^2}{x^2} + \frac{1}{x^2}}}$
 $\frac{1}{\sqrt{1 + 0}}$
 1

11. $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}$
 $\lim_{x \rightarrow -\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2 + 1}}{-\sqrt{x^2}}}$
 $\lim_{x \rightarrow -\infty} \frac{1}{-\sqrt{1 + \frac{1}{x^2}}}$
 $-\frac{1}{\sqrt{1 + 0}}$
 -1

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$$12. \lim_{x \rightarrow -\infty} \frac{(3x + \sqrt{9x^2 - x})}{1} \left(\frac{3x - \sqrt{9x^2 - x}}{3x - \sqrt{9x^2 - x}} \right)$$

$$\lim_{x \rightarrow -\infty} \frac{9x^2 - (9x^2 - x)}{3x - \sqrt{9x^2 - x}}$$

$$\lim_{x \rightarrow -\infty} \frac{x}{3x - \sqrt{9x^2 - x}}$$

$$\lim_{x \rightarrow -\infty} \frac{\frac{x}{x}}{\frac{3x}{x} - \frac{\sqrt{9x^2 - x}}{\sqrt{x^2}}}$$

$$\lim_{x \rightarrow -\infty} \frac{1}{3 + \sqrt{9 - \frac{1}{x}}}$$

$$\frac{1}{3 + \sqrt{9 - 0}} = \boxed{\frac{1}{6}}$$

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HOMEWORK

pg 245 - 246; 27 - 37, 49, 51, 53

and

Worksheet 4.5

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